Year 12 Lecture Series
Topic 6 – Integration

Inspired Learning
Mr A S Gill

Topic 6 - Integration

- Indefinite/Definite integration
- Finding the equation of a curve
- Area under a curve
- Area between a curve and a straight line

Prerequisites:

Topic 1 – Algebra
Topic 2 – Coordinate Geometry
Topic 4 - Differentiation

Topic 6 - Integration

Indefinite Integration

$$For\ f(x) = kx^n + c$$

$$f'(x) = n(kx^{n-1})$$

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$$\int 2x\,dx$$

$$\int 2x(x-5)\,dx$$

$$\int \frac{1}{x^5}\,dx$$

$$\int 7x^{\frac{5}{2}}\,dx$$

$$\int \frac{2x^2+3}{\sqrt{x}}\,dx$$

$$\int \frac{x+1}{\sqrt[3]{x}}\,dx$$

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Finding the equation of a curve

$$y - y_1 = m(x - x_1)$$

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$A\ curve\ passes\ through\ the\ point\ (1, 12) and\ its\ gradient\ function\ is\ (3x - 1)(x + 3). Find\ the\ equation\ of\ the\ curve.$

$The\ gradient\ function\ of\ a\ curve\ is\ \frac{dy}{dx} = 2x - 6.$

$At\ the\ point\ where\ the\ gradient\ is\ 0, the\ value\ of\ y\ is\ 1. Find\ the\ equation\ of\ the\ curve.$

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A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of f(1). [EDEXCEL, GCE Mathematics, C1, Jan 2012]

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A curve has equation $y = f(x)$. The point P with coordinates $(9, 0)$ lies on the curve. Given that

$$f'(x) = \frac{x+9}{\sqrt{x}}, \qquad x > 0$$

a find $f(x)$.

b Find the x-coordinates of the two points on $y = f(x)$ where the gradient of the curve is equal to 10. [EDEXCEL, GCE Mathematics, C1 1R, June 2013]

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Definite Integration

$$\int f'(x)dx = f(x) + c$$

$$\int_a^b f'(x)\,dx = [f(x)]_a^b = f(b) - f(a)$$

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$$\int_{3}^{5} 2x\,dx$$

$$If \int_{1}^{a} (2x-1)dx = 12, find\ the\ positive\ value\ of\ a.$$

$$\int_{4}^{6} \frac{1}{x^5}\,dx$$

$$\int_{1}^{4} \frac{2x^2+3}{\sqrt{x}}\,dx$$

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$$\int_3^5 2x\,dx$$

$$\int_4^6 \frac{1}{x^5}\,dx$$

$$\int_1^4 \frac{2x^2+3}{\sqrt{x}}\,dx$$

$$If \int_1^a (2x-1)dx = 12, find\ the\ positive\ value\ of\ a.$$

i Find $\int 24x^{-3}\,dx$.

ii Given that $\int_a^\infty 24x^{-3}\,dx = 3$, find the value of the positive constant a.

[OCR, GCE Mathematics, C2, June 2013]

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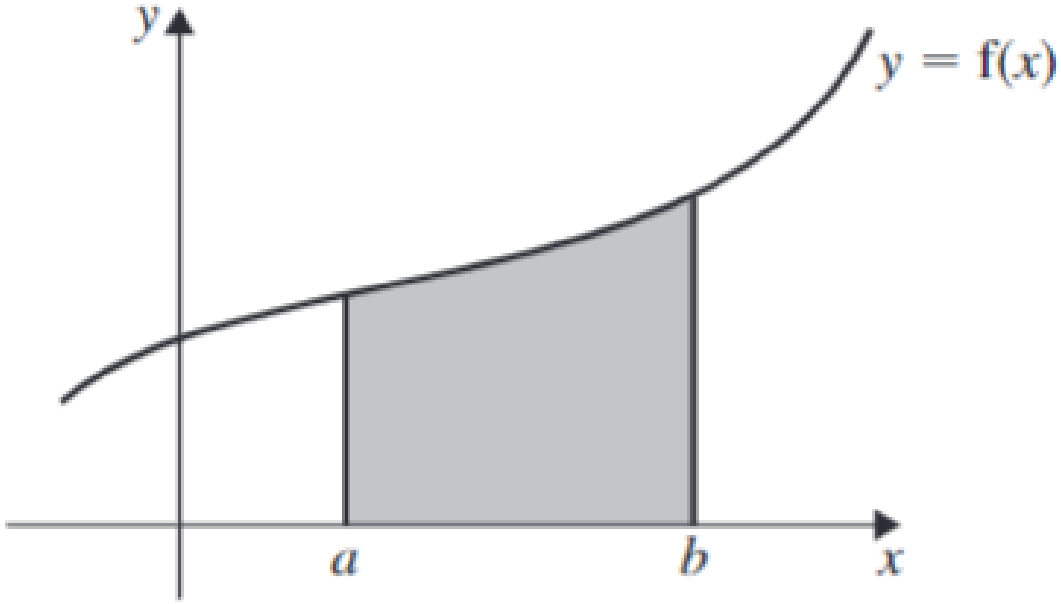
$Area\ between\ curve\ and\ x - axis$



$$\int_a^b f'(x)\,dx$$

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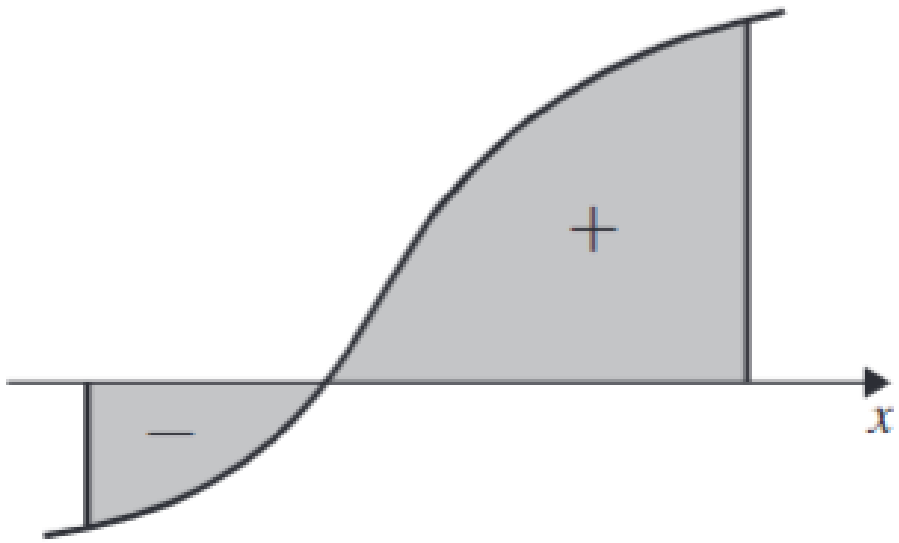
Important Note!



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Find the coordinates of the points where the curve $y = x^2 - 6x + 5$ *cuts the* $x - axis$, *and the area enclosed between the curve and the* $x - axis$.

Find the coordinates of the points where the curve $y = x^2 - 4$ *cuts the* $x - axis$.
Work out the area between the curve and the $x - axis$ *between* $x = 0$ *and* $x = 3$.

a Find $\int \left(x + 3 + \frac{8}{x^4}\right) dx$.

b Hence find the area of the region bounded by the curve C, the x-axis and the lines $x = 1$ and $x = 2$. [AQA, GCE Mathematics, C2, Jan 2011]

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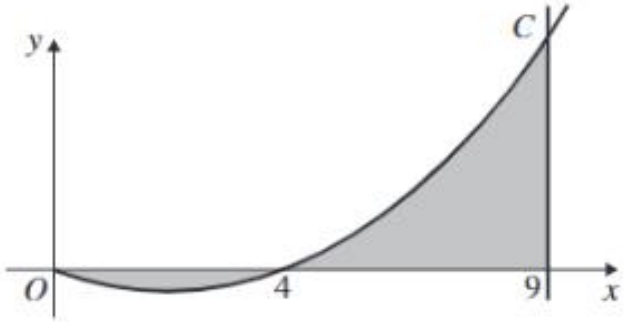
a Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.



The figure shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$.

b Use your answer from part **a** to find the total area of the shaded regions.

[EDEXCEL, GCE Mathematics, C2, June 2015]

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Area between curve and a straight line

- *Find coordinates using Simultaneous Equations*
- *Sketch*
- *Integrate*

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Find the points where the line $y = x$ *cuts the curve* $y = 4x - x^2$. *Calculate the area enclosed by the curve and the line* $y = x$.

Find the area enclosed between the curve $y = x - x^2$ *and the line* $y = \frac{x}{2}$.

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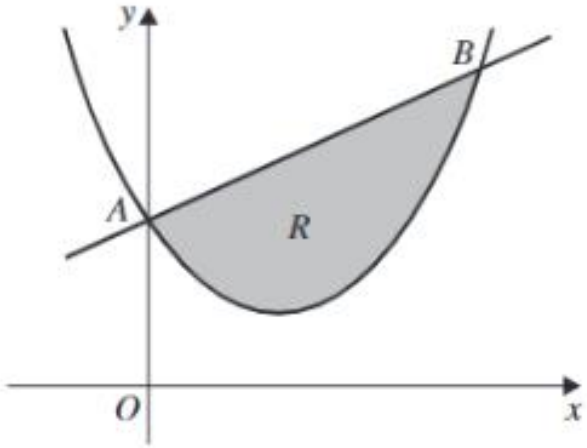
The curve C with equation $y = x^2 - 3x + 5$ and the straight line $y = x + 5$ intersect at the point $A(0, 5)$ and at the point B, as shown in the diagram below.



a Find the coordinates of the point B.

b Find $\int (x^2 - 3x + 5)\, dx$.

c Find the area of the shaded region R bounded by the curve C and the line segment AB. [AQA, GCE Mathematics, C1, June 2012]

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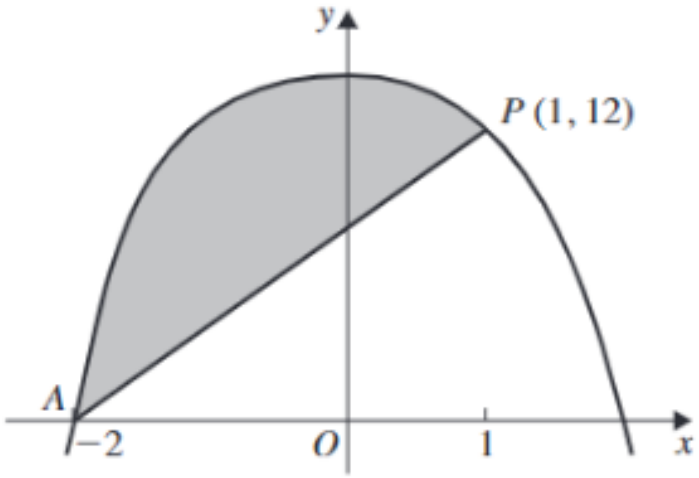
The curve sketched below passes through the point $A(-2, 0)$.



The curve has equation $y = 14 - x - x^4$ and the point $P(1, 12)$ lies on the curve.

a Find $\int_{-2}^{1} (14 - x - x^4)\, dx$.

b Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP. [AQA, GCE Mathematics, C1, Jan 2011]

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Summary:

- $Integration\ is\ defined\ as\ the\ opposite\ of\ differentiation.$
- $Always\ remember\ '+c'!$
- $Sketch\ out\ graphs\ when\ finding\ areas.$

See you next week!

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